Carnegie Mellon University
21-122, Summer Session 2012

Third Homework, due July 23th

1. Solve the following differential equations:

   I) $\dfrac{du}{dt} = 2 + 2u + t + tu$

   II) $(1 + \tan y)y' = x^2 + 1$

   III) $\dfrac{dy}{dx} = y^2 + 1$, $y(1) = 0$

   IV) $y' + 2y = 2e^x$

   V) $xy' + y = \sqrt{x}$

   VI) $(x^2 - 2y^2)dx + xydy = 0$

   VII) $xy' = y + 2xe^{-y/x}$

   VIII) $y' + y = \dfrac{1}{1 + e^{2x}}$

   IX) $y' = \dfrac{y^3}{1 - 2xy^2}$, $y(0) = 1$

   X) $\dfrac{dy}{dx} = \dfrac{x^3 - 2y}{x}$

2. The half-time of cesium-137 is 30 years. Suppose you have a 100mg sample.

   a) Find the mass that remains after 100 years

   b) How much of the sample remains after 100 years?

   c) After how long will only 1mg remain?

3. A tank initially contains 1000 liters of a mix of water and salt. The initial amount of salt is 10Kg. Water having a concentration of 0,1Kg/lt is introduced into the tank at a rate of $20lt/min$. The substance in the tank is mixing constantly, so that it is always homogenous, and flows out at the same rate ($20lt/min$). Let $Q(t)$ be the amount of salt in the tank at time t seconds. Write the equation describing the process and solve for $Q(t)$.

4. A curve in the first quadrant begins at the origin. The curve is so that the area below it between $(0, 0)$ and (x, y), equals a third of the area of the rectangle that has those points as opposite vertices. Find the equation of this curve.

5. Solve the following differential equation: $xy^2y' + y = x\cos(x)$.

6. Determine wether the series is convergent or divergent. If it is convergent find its sum.

   $a)$ $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$

   $b)$ $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+100}$

7. Find the values x for which the series converges. Find the sum for those values of x.

   $a)$ $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{3^n}$

   $b)$ $\displaystyle\sum_{n=0}^{\infty} \frac{\cos^n x}{2^n}$